Viscosity of High Energy Nuclear Fluids

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Relativistic high energy heavy ion collision cross sections have been interpreted in terms of almost ideal liquid droplets of nuclear matter. The experimental low viscosity of these nuclear fluids have been of considerable recent quantum chromodynamic interest. The viscosity is here discussed in terms of the string fragmentation models wherein the temperature dependence of the nuclear fluid viscosity obeys the Vogel-Fulcher-Tammann law.

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I. INTRODUCTION

There has been considerable interest in probing experimentally a theoretical quark-gluon plasma phase transition by employing ultra-relativistic two heavy nuclei scattering. It is not presently clear whether or not a deconfined plasma has actually been observed. However, such scattering experiments have apparently produced nuclear matter at very high energy density wherein the matter acts as a relativistic almost *ideal* fluid[1, 2, 3, 4]; i.e. a quark-gluon matter fluid with very low viscosity[5, 6, 7, 8].

In previous work[9, 10], we argued from the string fragmentation model, that a high energy density fluid may exhibit a glass-like behavior with meson and baryon strings playing a role analogous to polymers in some condensed matter glasses. At very high temperatures, the viscosity of the stringy fluid is actually quite small. It is this regime (of an almost ideal fluid) that is of central importance in ultra-relativistic heavy ion scattering. Our purpose is to study how the viscosity of resulting nuclear matter varies with energy density, pressure and baryon density.

In Sec.II we review the Kubo formula for the frequency dependent viscosity of a fluid and derive sum rules which allow for the determination of the zero frequency transverse viscosity in terms of the relaxation time for the non-diagonal elements of the pressure tensor. In Sec.III, we consider the quantized unit of circulation which contributes to turbulent eddy currents in the fluid. A bound for the Reynolds number of quantum turbulence leads to a quantum lower bound for the viscosity of the type that has been previously conjectured[11]. We review the low energy well established liquid drop model in Sec.IV and argue that the low energy density nuclear fluid is highly viscous. We turn to high energy nuclear matter in Sec.V and in particular discuss the formal conformal symmetry associated with glue. This system is thought to describe high energy QCD nuclear matter in the limit of low baryon density. In Sec.VI we review the notion of the QCD inspired string model and employ in Sec.VII the string fragment ion picture to compute the viscosity of high temperature nuclear matter. The viscosity reaches small values in the ultra high temperature limit. In the concluding Sec.VIII, we compare the string fragmentation (jet) picture with the quark gluon plasma view of high temperature nuclear matter.

II. PRESSURE FLUCTUATIONS

For an isotropic fluid in thermal equilibrium, the off diagonal elements of the pressure tensor (say $P_{xy}(\mathbf{r},t)$) fluctuations are described by the correlation function,

$$K(\mathbf{r}-\mathbf{r}',t) = \frac{1}{\hbar}\int_0^\beta \left\langle P_{xy}(\mathbf{r},t)P_{xy}(\mathbf{r}',-i\lambda)\right\rangle d\lambda, \tag{1}$$

wherein $\beta = (\hbar/k_BT)$. The complex frequency dependent fluid shear viscosity $\eta(\zeta)$ is determined by the Kubo formalism[12, 13, 14, 15, 16]

$$\mathcal{G}(t) = \int K(\mathbf{r},t)d^3\mathbf{r},$$
$$\eta(\zeta) = \int_0^\infty \mathcal{G}(t)e^{i\zeta t}dt, \quad \Im m\ \zeta > 0. \tag{2}$$

With ρ as the mass density of the fluid and c_∞ as the velocity of a finite high frequency transverse sound, one finds the dispersion relations

$$\mathcal{G}(t) = \frac{2}{\pi}\int_0^\infty \Re e\eta(\omega+i0^+)\cos(\omega t)d\omega,$$
$$\eta(\zeta) = -\frac{2i\zeta}{\pi}\int_0^\infty \frac{\Re e\eta(\omega+i0^+)d\omega}{\omega^2-\zeta^2}, \tag{3}$$

together with the sum rule

$$\mathcal{G}(0) \equiv \rho c_\infty^2 = \frac{2}{\pi}\int_0^\infty \Re e\eta(\omega+i0^+)d\omega. \tag{4}$$

In the low frequency limit

$$\eta \equiv \lim_{\omega \to 0} \Re e \eta(\omega + i0^+) = \int_0^\infty \mathcal{G}(t)dt, \tag{5}$$

the relaxation time τ for the decay of the correlation function

$$\tau = \frac{1}{\mathcal{G}(0)} \int_0^\infty \mathcal{G}(t)dt = \frac{1}{\rho c_\infty^2}\ \eta \tag{6}$$

uniquely determines the shear viscosity via[17, 18, 19, 20]

$$\eta = \rho c_\infty^2 \tau. \tag{7}$$

To find the viscosity at zero frequency for a fluid of mass density ρ employing the Kubo formalism, one must determine the velocity of transverse sound c_∞ and the relaxation time τ of the off diagonal elements of the pressure.

III. QUANTUM TURBULENCE

The following arguments, first applied by Feynman[21] to superfluid Helium, hold true for any fluid whose constituent particles all have mass M. For flows with eddy currents on a velocity scale V and a length scale L, the *circulation* of the eddy velocity field is given by

$$\oint \mathbf{v} \cdot d\mathbf{r} = VL \tag{8}$$

From quantum fluid mechanics, the Feynman number $\mathcal{F}$ of quantum vortices in a circulating eddy is given by the Bohr quantization of circulation

$$M \oint \mathbf{v} \cdot d\mathbf{r} = \oint \mathbf{p} \cdot d\mathbf{r} = 2\pi\hbar\mathcal{F}; \tag{9}$$

i.e. the Feynman number is

$$\mathcal{F} = \frac{MVL}{2\pi\hbar}. \tag{10}$$

On the other hand, the Reynolds number for a circulating eddy is well known to be

$$\mathcal{R} = \frac{\rho VL}{\eta}, \tag{11}$$

yielding a ratio which depends only on intrinsic fluid properties and not on the nature of the eddy flow;

$$\frac{\mathcal{F}}{\mathcal{R}} = \frac{M\eta}{2\pi\hbar\rho}. \tag{12}$$

If $\rho = Mn$, with n as the number of constituent particles per unit volume, then

$$\frac{\mathcal{F}}{\mathcal{R}} = \frac{\eta}{2\pi\hbar n} \quad (\mathrm{non-relativistic}). \tag{13}$$

Since the Feynman number of quantized vortices in an eddy is larger than the Reynolds number of the eddy, we have the quantum viscosity inequality

$$\mathcal{F} > \mathcal{R} \quad \Rightarrow \quad \eta > 2\pi\hbar n. \tag{14}$$

The above lower bound is found to apply to all known chemically pure substances. For example, for pure water $\mathcal{F} \approx 45\mathcal{R} \gg \mathcal{R}$.

IV. OLD LIQUID DROP MODEL

The first nuclear fluid model was employed in a non-relativistic context. In the liquid drop model, one has a momentum sphere of filled quasi-particle states out to a Fermi momentum $p_F = \hbar k_F$. The number of nucleons A in a spherical nucleus of radius R can be computed by filling up a Fermi sphere in momentum space and a sphere in position space

$$A = \frac{4}{(2\pi)^3}\left(\frac{4\pi k_F^3}{3}\right)\left[\frac{4\pi R^3}{3}\right] = \frac{8(k_F R)^3}{9\pi}. \tag{15}$$

The liquid drop nucleus is nearly incompressible so that the radius obeys

$$R = aA^{1/3} \quad \mathrm{wherein} \quad a \approx 1.2 \times 10^{-13}\mathrm{cm}. \tag{16}$$

The Fermi velocity for a nearly ideal Fermi fluid is given by

$$v_F = \frac{1}{\hbar}\frac{dE_F}{dk_F} \approx \frac{\hbar k_F}{M} = \left(\frac{9\pi}{8}\right)^{1/3} \frac{\hbar}{Ma}\ . \tag{17}$$

Note that $(v_F/c) \approx 0.27$ which implies that the nucleons within the droplet are not entirely non-relativistic.

The density of energy states per unit volume at the Fermi surface is given by

$$g_F = \frac{4}{(2\pi)^3}\left(4\pi k_F^2 \frac{dk_F}{dE_F}\right) = \frac{2}{\pi^2}\left(\frac{k_F^2}{\hbar v_F}\right). \tag{18}$$

The specific heat per unit volume of a Fermi liquid is given by

$$c = T\frac{ds}{dT} = \left(\frac{\pi^2 k_B^2 g_F}{3}\right)T + \dots \quad \mathrm{as} \quad T \to 0. \tag{19}$$

Which implies a low temperature entropy per unit volume

$$s \approx \left(\frac{\pi^2 k_B^2 g_F}{3}\right)T = \left(\frac{\pi^2 k_B^2 g_F}{3}\right)\frac{d\epsilon}{ds} \tag{20}$$

wherein ϵ is the excitation energy per unit volume. For weakly excited states the entropy per unit volume obeys

$$s = k_B\left(\frac{2\pi^2 g_F}{3}\right)^{1/2}\sqrt{\epsilon} = 2k_B k_F\sqrt{\frac{\epsilon}{3\hbar v_F}}\ . \tag{21}$$

In terms of the total droplet entropy and energy, the thermodynamic equation of state for excitation energy E weakly above the ground state energy E_{0A} is given by

$$\frac{S(E)}{k_B} = \frac{4}{3} A \left(\frac{\pi k_F^2 a^3}{\hbar v_F} \right)^{1/2} \sqrt{\frac{E - E_{0A}}{A}} \,. \tag{22}$$

The density of excited energy levels for the droplet as a whole then reads

$$D(E) \approx D(E_{0A}) \exp \left[\frac{S(E)}{k_B} \right], \tag{23}$$

which is experimentally reasonably accurate for heavy nuclei; a logarithmic plot of the density of excited states above the ground state yields a square root behavior[22],

$$\ln \left[\frac{D(E)}{D(E_{0A})} \right] \approx \sqrt{\frac{E - E_{0A}}{\tilde{E}_A}} \quad \text{as} \quad E \to E_{0A} + 0^+. \tag{24}$$

In the opposite limit $E \gg E_{0A}$, one finds $S(E) \approx E/T_0$ wherein T_0 is the Hagedorn temperature[23].

The viscosity of the Fermi fluid in the liquid drop is quite high. This may be understood by writing Eq.(7) in the form

$$\eta = v_F^2 \tau_c. \tag{25}$$

wherein τ_c is an appropriate collision time for quasi-particle scattering. From two body scattering phase space considerations, one finds that $\tau_c \propto T^{-2}$ which implies a low nuclear energy liquid drop viscosity given by

$$\eta \approx K\hbar \left(\frac{m v_F}{\hbar} \right)^5 \left(\frac{\hbar v_F}{k_B T} \right)^2 \tag{26}$$

in which K is a dimensionless quantity of order unity, depending on the angular variation of the two body cross section. At very low temperatures, the viscosity grows very high. It will be discussed in what follows that as the temperature grows ever larger, the viscosity diminishes to smaller values until it is quantum limited and the fluid is nearly ideal.

V. GLUE AND CONFORMAL SYMMETRY

If $G^a_{\mu\nu}$ denotes the gluon field,

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \left(\frac{g}{\hbar c} \right) f^a_{bc} A^b_\mu A^c_\nu, \tag{27}$$

then the action describing pure glue reads

$$W = -\frac{1}{2c} \int G^a_{\mu\nu} G^{\mu\nu}_a d^4x. \tag{28}$$

Under the conformal scale changes in space time

$$\begin{aligned} x \to \bar{x} &= \lambda x, \\ A^a_\mu(x) \to \bar{A}^a_\mu(\bar{x}) &= \frac{1}{\lambda} A^a_\mu(x), \\ G^a_{\mu\nu}(x) \to \bar{G}^a_{\mu\nu}(\bar{x}) &= \frac{1}{\lambda^2} G^a_{\mu\nu}(x), \end{aligned} \tag{29}$$

the action is left invariant; i.e. $W \to \bar{W} = W$. Such symmetry dictates that the trace of the energy-pressure tensor

$$-\Theta = T^\mu_{\ \mu} = 3P - u \tag{30}$$

is null. In Eq.(30), u represents the energy per unit volume and P represents the pressure. Formally, the energy-pressure tensor for glue is given by

$$T^{Glue}_{\mu\nu} = \left(g^{\lambda\sigma} G^a_{\mu\sigma} G_{a\nu\lambda} - \frac{1}{4} g_{\mu\nu} G^{a\mu\sigma} G_{a\mu\sigma} \right), \tag{31}$$

which implies $\Theta = 0$ and which formally verifies the conformal invariance properties

$$-\Theta^{Glue} = T^{\mu\ Glue}_{\ \mu} = 0. \tag{32}$$

On the other hand, for a theory with massive quarks the trace of the energy-pressure tensor is

$$\Theta^{Quark} = \sum_f m_f c^2 \bar{q}_f q_f. \tag{33}$$

In the presence of Gluons, the trace of the energy-pressure tensor can reassert its strength via

$$\Theta = \left\langle \sum_k m_k c^2 \bar{q}_k q_k \right\rangle. \tag{34}$$

For example, for a heavy quark, say Q with mass M, in the presence of a low energy gluon field configuration one finds the trace anomaly[24]

$$\Theta_Q = Mc^2 \left\langle 0; G \left| \bar{Q} Q \right| 0; G \right\rangle = -\left(\frac{2\alpha_s}{3\pi} \right) G^a_{\mu\nu} G^{\mu\nu}_a, \tag{35}$$

where $\alpha_s = g^2/4\pi\hbar c$ is the QCD strong coupling strength. Employing purely thermodynamic reasoning with the Gibbs-Duhem equation,

$$dP = s dT + \sum_f n_f d\mu_f, \tag{36}$$

and the Euler relation for the energy density

$$u = Ts - P + \sum_f \mu_f n_f, \tag{37}$$

one finds the trace of the energy-pressure tensor

$$\begin{aligned} \Theta &\equiv u - 3P, \\ \Theta &= Ts - 4P + \sum_f \mu_f n_f, \\ \Theta &= T \left(\frac{\partial P}{\partial T} \right)_\mu - 4P + \sum_f \mu_f \left(\frac{\partial P}{\partial \mu_f} \right)_{T, \mu_{k \neq f}}. \end{aligned} \tag{38}$$

If one replaces the quark chemical potentials with the quark activities

$$z_f \equiv \exp(\mu_f / k_B T), \tag{39}$$

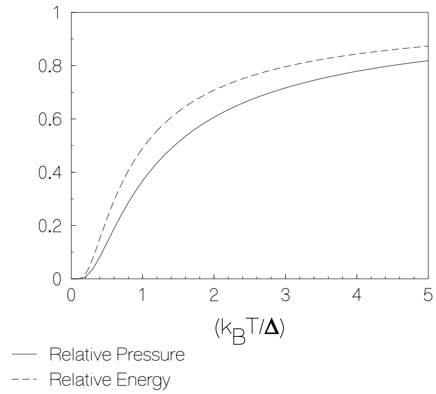

FIG. 1: Shown are the relative pressure P/P_∞ and relative energy u/u_∞ plotted as a function of temperature.

then the thermodynamic identity for the trace of the energy-pressure tensor has the more simple form

$$\Theta = T\left(\frac{\partial P}{\partial T}\right)_z - 4P, \tag{40}$$

or equivalently, the energy density is related to the pressure according to

$$u = T\left(\frac{\partial P}{\partial T}\right)_z - P. \tag{41}$$

Consider the following simple model of a proposed quark-gluon plasma deconfinement. At very high temperatures, one considers the quarks and gluons to act as a gas exhibiting conformal symmetry. This implies a pressure P_∞ and an energy density u_∞ obeying

$$u_\infty = 3P_\infty = \text{const.}\left[\frac{(k_BT)^4}{(\hbar c)^3}\right]. \tag{42}$$

At low temperatures this ideal behavior cannot persist and the conformal symmetry is broken by an energy gap Δ. The pressure may then be suppressed by a Boltzmann factor,

$$P = P_\infty e^{-\Delta/k_BT}, \tag{43}$$

which, in virtue of Eq.(41), implies

$$u = u_\infty e^{-\Delta/k_BT}\left[1 + \frac{\Delta}{3k_BT}\right]. \tag{44}$$

Plots of the quark-gluon deconfinement based on this model are shown in FIG. 1. This simple model of the crossover between the confined phase and the quark-gluon plasma phase is in qualitative agreement with lattice QCD computations[25]. The theoretical equilibrium statistical thermodynamic arguments presented here are not directly applicable to ultra high energy heavy ion nuclear collision for two reasons: (i) Very energetic collisions involve non-equilibrium processes and (ii) the collision fragments are most often described by QCD string fragmentation models[26].

VI. LUND STRING MODEL

To compute non-equilibrium processes wherein quarks are connected to one another via strings (gluon electric flux tubes) we begin with the activation entropy $S_1(E)$ of a single string; It is[9]

$$\frac{S_1}{k_B} = \left(\frac{E}{k_BT_0}\right) - \frac{7}{2}\ln\left[\frac{E}{k_BT_0}\right] + \frac{\tilde{S}}{k_B}\,,$$
$$\frac{\tilde{S}}{k_B} = \frac{1}{2}\left[\ln(3) + 7\ln\left(\frac{2\pi}{3}\right)\right], \tag{45}$$

wherein the Hagedorn temperature T_0 is related to the string tension σ via

$$k_BT_0 = \sqrt{\frac{3\hbar c\sigma}{4\pi}} \approx 207\ \text{MeV}. \tag{46}$$

The numerical value of σ (and thereby T_0) is found from the experimental slope of the lowest meson Regge trajectory. The energy of a single string as a function of temperature may be computed via the microcanonical statistical thermodynamic law,

$$\frac{1}{T} = \frac{dS_1(E)}{dE}, \tag{47}$$

yielding,

$$E = \Phi\left[\frac{T}{T - T_0}\right] \quad \text{where} \quad \Phi = \frac{7k_BT_0}{2} \approx 725\ \text{MeV}. \tag{48}$$

The plot of energy as a function of temperature is given in FIG. 2. Note that the energy always decreases as the temperature increases. The heat capacity is negative in the physically allowed string temperature range; i.e.

$$C = \frac{dE}{dT} = -\frac{\Phi}{T_0}\left[\frac{T_0}{T - T_0}\right]^2 < 0,$$
$$0 > T >= -\infty \quad \text{and} \quad \infty > T > T_0. \tag{49}$$

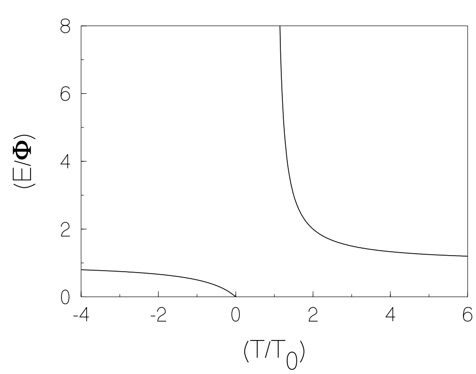

FIG. 2: Shown is the energy of a single Lund string as a function of temperature. Note that the heat capacity of the string $C = (dE/dT) < 0$ in the physical but metastable temperature ranges $-\infty < T < 0$ and $T_0 < T < \infty$.

The physical interpretation of Eqs.(49) is worthy of note. Since in accordance with the second law of thermodynamics, stable systems have a positive heat capacity we thereby deduce that a single string *during a collision* is in a metastable state.

As discussed in previous work[9, 10], the string near the beginning of the collision has a very low energy $0 < E < \Phi$. Thus, the initial string temperature is negative. The string *cannot* have negative energy which implies that string temperatures between zero and the Hagedorn temperature are not allowed; i.e. $0 < T < T_0$ is unphysical. As the energy of the string energy increases due to the absorption of collision energy $E \to \Phi - 0^+$, the temperature decreases $T \to -\infty$. When only a little bit more collision energy is absorbed by the string $E = \Phi + 0^+$ the string temperature is virtually a positive infinite value. Any further cooling of the temperature requires more and more addition of the collision energy to be absorbed by a single string. We note in passing, that the notion of a negative temperature going positive by first approaching $-\infty$ and reappearing at $+\infty$ is *not* new. This property experimentally appears in laboratory nuclear magnetic resonant spin systems and laser media with inverted energy populations.

For energy slightly above Φ, the string temperature is virtually infinite; i.e. $T \to +\infty$ as $E \to \Phi + 0^+$. The question then arises (in the ultra high collision energy per nucleon limit) as to how much collision energy can be transferred to a single string in order to bring the temperature down from its very high temperature state. As will be made clear in the next Sec.VII, the cooling time exponentially grows larger with decreasing temperature. Furthermore, the higher the collision energy per nucleon, the shorter the duration of a collision. When the duration of the collision is comparable to the cooling time for a given hot string temperature, the temperature stops getting smaller. The string temperatures stay high. While it is certainly possible to create many new strings with the ultra high collision energy, e.g. mesons, it will not be easy to cool the strings down from having a very high temperature. Let us now be more explicit regarding the time-scales in a stringy liquid droplet; i.e. the *new* high energy liquid droplet "pancake" model.

VII. LOW VISCOSITY AND THE IDEAL FLUID

The role of the entropy in determining transition rates follows from the the calculation rules of quantum mechanical transition rates; In particular one averages over initial states and sums over final states. In general the number of quantum states Ω is related to the entropy S by the Boltzman formula

$$S = k_B \ln \Omega. \tag{50}$$

The ratio of the number of final states to the number of initial states is then determined by the exponential of an entropy difference

$$\frac{\Omega_f}{\Omega_i} = \exp\left[\frac{S_f - S_i}{k_B}\right]. \tag{51}$$

Transition rates may be phase space dominated by the exponential entropy factors. The dominant entropy arises from bosonic gluon excitations of the QCD string. The Fermi or Bose nature of physical particles depend only on the number, respectively odd or even, of quarks tied together (in a polymer-like fashion) by the gluon color electric flux string. The thermal relaxation times for very hot $T \to +\infty$ strings to cool down to finite temperatures strings depends on the exponential of an entropy difference

$$\frac{\tau}{\tau_\infty} = \exp\left[\frac{S_1 - S_{1\infty}}{k_B}\right]. \tag{52}$$

From Eqs.(45) and (47) one finds

$$\frac{S_1}{k_B} = \frac{7}{2}\left(\frac{T_0}{T - T_0}\right) - \left(\frac{7}{2}\right)\ln\left[\frac{T}{(T - T_0)}\right] + \frac{S_{1\infty}}{k_B}\ , \tag{53}$$

wherein

$$\frac{S_{1\infty}}{k_B} = \frac{7}{2}\left[1 + \ln\left(\frac{2\pi}{3}\right) - \ln\left(\frac{7}{2}\right)\right] + \frac{1}{2}\ln 3. \tag{54}$$

Thus the cooling time scale from Eqs.(52) and (53) may be written as

$$\tau = \tau_\infty \left(\frac{T - T_0}{T}\right)^{7/2} \exp\left[\frac{\Phi}{k_B(T - T_0)}\right]. \tag{55}$$

The viscosity implicit in QCD inspired string models is of a form which follows from Eqs.(7) and (55); It is

$$\eta = \rho c_\infty^2 \tau_\infty \left(\frac{T - T_0}{T}\right)^{7/2} \exp\left[\frac{\Phi}{k_B(T - T_0)}\right]. \tag{56}$$

To compute the attempt frequency τ_∞^{-1} for a string to be created, for example between a quark anti quark pair, we need to consider the rate with which a quark in the "negative vacuum Dirac-Fermi sea" is excited into a positive energy state leaving behind a quark hole. The phase space element along the axis of the string connecting the quark and the hole is

$$d^2\tilde{N} = \frac{dp dx}{2\pi\hbar} = \frac{dp}{dt}\left(\frac{dx dt}{2\pi\hbar}\right), \tag{57}$$

wherein the rate of change of momentum of the quark is the force, i.e. the string tension σ. Thus, the attempt frequency per unit time per unit length to excite a quark anti quark string, i.e. meson, is

$$\frac{d^2\tilde{N}}{dt dx} = \frac{\sigma}{2\pi\hbar}. \tag{58}$$

For string of length L, the energy is $E = \sigma L$ yielding the attempt frequency

$$\frac{1}{\tau_\infty} = \frac{d\tilde{N}}{dt} = \frac{E}{2\pi\hbar} = \frac{\Phi}{2\pi\hbar}\left(\frac{T}{T-T_0}\right), \qquad (59)$$

wherein Eq.(48) has been invoked.

From Eqs.(56) and (59) follows a central result of this work; i.e. the high energy nuclear fluid viscosity

$$\eta = \eta_{ideal}\left(\frac{T-T_0}{T}\right)^{9/2}\exp\left[\frac{\Phi}{k_B(T-T_0)}\right], \qquad (60)$$

with the ideal quantum viscosity determined as

$$\eta \approx \eta_{ideal} \quad \text{if} \quad T \gg T_0,$$
$$\frac{\eta_{ideal}}{2\pi\hbar} = \bar{n},$$
$$\bar{n} = \frac{\rho c_\infty^2}{\Phi} \leq \frac{\rho c^2}{\Phi}\ . \qquad (61)$$

Eqs.(60) and (61) for the viscosity of a stringy QCD fluid phase is quite similar to the viscosity of a polymer chain glass. As the temperature is lowered from above to T_0, the viscosity becomes extraordinarily high and the fluid behaves more as a glass then as an ideal fluid. For the ultra high energy collisions of heavy nuclei, the duration of the collision is sufficiently short that $T \gg T_0$. For such a range of hot string temperatures, the fluid viscosity is extraordinarily low, i.e. the viscosity is the quantum limited value $\eta \approx \eta_{ideal}$. The QCD fluid phase is thereby ideal.

VIII. CONCLUSION

We have derived Kubo formula sum rules which allowed for the determination of the transverse viscosity in terms of the relaxation time of the pressure tensor. We then considered the quantized unit of circulation which contributes to turbulent eddy currents in the fluid. If the the quantum turbulent Feynman number $\mathcal{F}$ is to exceed classical turbulent Reynolds number $\mathcal{R}$, then there exists a quantum lower bound to viscosity which has been previously conjectured[11]. Nuclear matter, as it appears in ultra high energy heavy nuclei scattering, was thought by some workers to have the formal conformal symmetry associated with deconfined glue at low baryon density. Here, the string fragmentation model was employed wherein the quarks and anti quarks are connected to one another by strings whose electric flux tubes consist of glue. The notion of a low baryon density nuclear fluid is thereby defined as a very stringy gluon liquid with not so many quarks and anti quarks. This stringy liquid is not quite the same as a deconfined quark gluon plasma. The viscous properties of a QCD stringy liquid is as described by Eqs.(60) and (61). The stringy liquid is closely analogous to a polymer glass. While the viscosity grows quite high as the glass (Hagedorn) temperature is approached from above, for temperatures well above the glass temperature ($T \gg T_0$), the viscosity is quite small and quantum limited. These small viscosity values imply and almost ideal ultra high energy nuclear fluid.

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